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Individual Cylinder air/fuel ratio observer on IC engine using Takagi-Sugeno's fuzzy model

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Abstract—**The aim of this article is to design an estimator of the individual air-fuel ratio of each cylinder in an turbocharger internal combustion engine. A non linear model of the exhaust manifold is used to describe the dynamic of the different gas mass flows. Then, a Takagi-Sugeno's fuzzy model is derived from which an observer is developed. Finally, some simulations are given to show the efficiency of the proposed method.**

Keywords— **air fuel ratio control ; Turbocharger internal combustion ; TS fuzzy systems.**

I. INTRODUCTON

The control of the air-fuel ratio (AFR) still remains a key point for engine control and especially for pollution emission purpose. The AFR is defined as the quantity of air over the quantity of fuel injected in each cylinder compared with stoichiometric condition. This variable characterizes the quality of the combustion process, and the performances of the engine according to fuel consumption and emissions. The catalytic technology for exhaust gas imposes an AFR of 1 ($\pm 5\%$). Generally, for a gasoline engine, the quantity of fuel injected in each cylinder controls the AFR. The main problem is that this variable is measured in the exhaust manifold. A way to control precisely the consumption and the pollution emission should be to obtain the AFR of each cylinder, which is the goal of our work.

[1, 2, 3, 4]. Their objective is to ensure a fast AFR regulation around 1 even during fast transient phases.
The literature dealing with the AFR control is important [5, 6], and another proposed solution uses an accurate estimation of the AFR for each cylinder [7, 8, 9, 10]. which allows an AFR regulation cylinder by cylinder.
Of course, to derive such control laws accurate models are required. A way to solve this problem is to use a real time estimation of the AFR estimation for each cylinder via an observer.

According to these preliminary remarks, the goal of this paper is to propose an estimation of individual AFR able to cope with real time constraint. To achieve this goal, the first part of the paper presents the chosen model and its transformation into a Takagi-Sugeno (TS) model [11]. The second part deals with the design of an associated observer. The conditions obtained are sufficient but use the Linear Matrix Inequality (LMI) constraints [12]. Lastly, some simulation results are given to show the efficiency of the proposed method.

II. EXHAUST MANIFOLD MODELING

A. Exhaust gas dynamics

The considered model to describe the exhaust manifold comes from [13]. The following hypotheses are assumed. The exhaust manifold is represented as a volume in which the mass conservation principle holds. Moreover, the temperature inside is chosen as a constant.
The measured air-fuel ratio is linked to the mass of air around the lambda sensor and to the total mass of gas. Thus, applying the mass conservation principle in the exhaust manifold, the following equations are obtained:

$$Ne\frac{dM_T}{d\alpha} = \sum_{i=1}^{n_{cyl}} d_i(\alpha) - d_T(M_T) \tag{1}$$

$$Ne\frac{dM_{air}}{d\alpha} = \sum_{i=1}^{n_{cyl}} (1-\lambda_i) d_i(\alpha) - d_{air}(M_{air}) \tag{2}$$

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Table 1: Part list

Symbol	Quantity	
M_T	Total mass	kg
M_{air}	Air mass	kg
M_0	Total Mass at atmospheric conditions	kg
d_i	Gas mass flow from the cylinder i	kg/s
d_T	Gas mass flow into the turbine	kg/s
Ne	Engine speed	tr/mn
P_{ex}	Exhaust manifold pressure	bar
α	Crankshaft angle	
λ_g	measured AFR	
λ_i	reference AFR	
V_{ex}	Volume of the exhaust manifold	
T_{ex}	Temperature at the exhaust manifold	°K
R	Perfect gas constant	$J\ mol^{-1}\ k^{-1}$
n_{cyl}	Number of cylinder	

The turbocharger is modelled considering a gas flow through a restriction [14, 15]. The flow law throught the turbine d_T is a function of the total mass M_T with this expression:

$$d_T(M_T) = M_T \times p(M_T) \tag{3}$$

where:

$$p(M_T) = f(N_{turb})\sqrt{\frac{2g}{g-1}\left(\left(\frac{M_T}{M_0}\right)^{\frac{-2}{g}} - \left(\frac{M_T}{M_0}\right)^{\frac{-(g+1)}{g}}\right)}$$

and the air composition is supposed the same as the exhaust manifold. The air flow through the turbine is therefore:

$$d_{air}(M_{air}) = \frac{M_{air}}{M_T} d_T(M_T) = M_{air} \times p(M_T) \tag{4}$$

The gas flow from one cylinder d_i, $i \in \{1,\ldots,n_{cyl}\}$ is modelled according to the crankshaft angle (Figure 1). Then, using the periodicity property of this flow, the one of each cylinder is obtained.

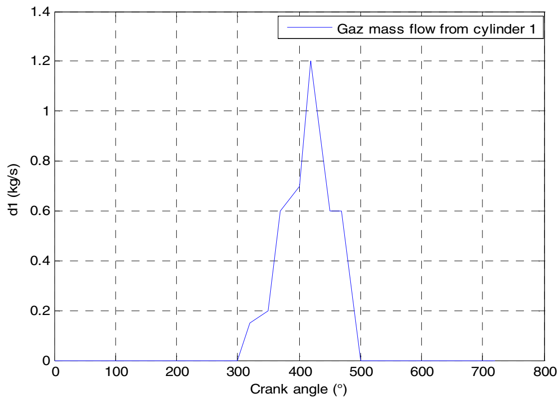

Figure 1. Gas flow from cylinder 1

So the equation (1) and (2) becomes:

$$\begin{cases} Ne\dfrac{dM_T}{d\alpha} = \displaystyle\sum_{i=1}^{n_{cyl}} d_i(\alpha) - M_T p(M_T) \\ Ne\dfrac{dM_{air}}{d\alpha} = \displaystyle\sum_{i=1}^{n_{cyl}} (1-\lambda_i) d_i(\alpha) - M_{air} p(M_T) \end{cases} \tag{5}$$

B. State-space representation

The following nonlinear state space representation of (5) is then obtained:

$$\begin{cases} \begin{bmatrix} \dot{M}_T \\ \dot{M}_{air} \end{bmatrix} = \begin{bmatrix} -f_1(M_T) & f_2(M_{air}) \\ 0 & f_2(M_{air}) - f_1(M_T) \end{bmatrix} \begin{bmatrix} M_T \\ M_{air} \end{bmatrix} - \begin{bmatrix} 0 \\ \sum_{i=1}^{n_{cyl}} \lambda_i d_i(\alpha) \end{bmatrix} \\ y = \begin{bmatrix} \gamma & 0 \\ f_3(M_T) & -f_3(M_T) \end{bmatrix} \begin{bmatrix} M_T \\ M_{air} \end{bmatrix} \end{cases} \tag{6}$$

with three nonlinear functions:

$$f_1(M_T) = p(M_T) \tag{7}$$

$$f_2(\alpha, M_{air}) = \frac{\sum_{i=1}^{n_{cyl}} d_i(\alpha)}{M_{air}} \tag{8}$$

$$f_3(M_T) = \frac{1}{M_T} \tag{9}$$

bounded as follows: $\underline{f_i} \le f_i(\cdot) \le \overline{f_i}$, for $i \in \{1,2,3\}$.

The considered outputs are:

P_{ex}, the pressure in the exhaust manifold related to the total mass by $P_{ex} = \gamma \times M_T$ with $\gamma = \frac{R \times T_{ex}}{V_{ex}}$, and λ_g, the global air fuel ratio measurement given by a AFR sensor situated downstream the turbine and expressed as:

$$\lambda_g = 1 - \frac{M_T}{M_{air}} \tag{10}$$

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C. TS model associated

First of all, let us recall that TS fuzzy models allow representing exactly nonlinear models in a compact set of the state variables [16]. A way to derive such TS models is to use the so-called sector nonlinearity approach. It consists in representing a bounded nonlinearity, i.e. $\underline{f_i} \le f_i(\cdot) \le \overline{f_i}$ using two functions verifying the convex sum property:

$$\forall i \in \{1,2,3\}, w_i^1(t) = \frac{\overline{f_i} - f_i(.)}{\overline{f_i} - \underline{f_i}} = 1 - w_i^2(t) \qquad (11)$$

Finally, using the following weighted functions:

$$h_1 = w_1^1 w_2^1 w_3^1,\ h_2 = w_1^1 w_2^1 w_3^2,\ h_3 = w_1^1 w_2^2 w_3^1,$$
$$h_4 = w_1^1 w_2^2 w_3^2,\ h_5 = w_1^2 w_2^1 w_2^1,\ h_6 = w_1^2 w_2^1 w_3^2,$$
$$h_7 = w_1^2 w_2^2 w_3^1,\ h_8 = w_1^2 w_2^2 w_3^2 \qquad (12)$$

A eight rules TS model is obtained as follows:

$$\begin{cases} N_e \dot{x}(t) = \sum_{i=1}^{8} h_i(z(t)) A_i x(t) + \theta(t) \\ y(t) = \sum_{i=1}^{8} h_i(z(t)) C_i x(t) \end{cases} \qquad (13)$$

with the state vector $x(t) = [M_T \quad M_{air}]^T$, the output vector $y(t) = [P_{ex} \quad \lambda_g]^T$, the vector of the unknown parameters $\theta(t) = \begin{bmatrix} 0 & \sum_{i=1}^{n_{cyl}} \lambda_i d_i \end{bmatrix}^T$ and the matrices of the model:

$$A_1 = A_5 = \begin{bmatrix} -\underline{f_1} & \underline{f_2} \\ 0 & \underline{f_2} - \underline{f_1} \end{bmatrix}, A_2 = A_6 = \begin{bmatrix} -\underline{f_1} & \overline{f_2} \\ 0 & \overline{f_2} - \underline{f_1} \end{bmatrix}$$

$$A_3 = A_7 = \begin{bmatrix} -\overline{f_1} & \underline{f_2} \\ 0 & \underline{f_2} - \overline{f_1} \end{bmatrix}, A_4 = A_8 = \begin{bmatrix} -\overline{f_1} & \overline{f_2} \\ 0 & \overline{f_2} - \overline{f_1} \end{bmatrix}$$

$$C_1 = C_3 = C_5 = C_7 = \begin{bmatrix} \gamma & 0 \\ \underline{f_3} & -\underline{f_3} \end{bmatrix}, \text{ and}$$

$$C_2 = C_4 = C_6 = C_8 = \begin{bmatrix} \gamma & 0 \\ \overline{f_3} & -\overline{f_3} \end{bmatrix}.$$

III. ESTIMATION OF THE INDIVIDUAL AFR

A. TS observer design

In this part, a fuzzy observer design will be proposed. It allows to estimate at once the state vector and the unknown vector, authers works deals with the same thematic [17, 18, 19].

For sake of simplicity, let us define some notations. For any $h_i(z(t))$ some scalar positive functions satisfy the convex property and for some matrices $Y_i, i \in \{i,\dots,r\}$: we define $Y_z = \sum_{i=1}^{r} h_i(z(t)) Y_i$ and $Y_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) Y_{ij}$. At last, an $(*)$ indicates the transpose quantity in a matrix expression. For example $(A - KC) + (*)$ stands for $(A-KC)+(A-KC)^T$.

Let us consider the observer (14):

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}}(t) \\ \hat{\theta}^{(n)}(t) \end{pmatrix} = \begin{pmatrix} A_z & I \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{x}(t) \\ \hat{\theta}^{(n-1)}(t) \end{pmatrix} - \begin{pmatrix} L_z \\ K_z \end{pmatrix} (y(t) - \hat{y}(t)) \\ \hat{y}(t) = (C_z \quad 0) \begin{pmatrix} \hat{x}(t) \\ \hat{\theta}^{(n-1)}(t) \end{pmatrix} \end{cases} \qquad (14)$$

with $\hat{x}(t)$ the estimated state vector, $\hat{\theta}^{(n)}(t)$ the n time derivative of the estimated unknown parameter vector and $\hat{y}(t)$ the estimated of the output vector. The observer gains L_i and K_i for $i \in \{1,\dots,r\}$.

Let us define:

$$\Upsilon_{ij} = \left(\begin{bmatrix} P \begin{bmatrix} A_i & I & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} - P \begin{bmatrix} L_j \\ K_j \end{bmatrix} [C_i \quad 0 \quad \cdots \quad 0] \end{bmatrix} \right. \\ \left. + (*) + 2\alpha P \right) \qquad (15)$$

Theorem 1:

The observer (14) is globally asymptotically stable, if it exists a real scalar α, some matrices $P > 0$, L_i and K_i for $i \in \{1,\dots,r\}$ such that the following conditions hold for Υ_{ij} defined in (15):

$$\Upsilon_{ii} < 0 \qquad (16)$$

$$\Upsilon_{ij} + \Upsilon_{ji} < 0 \quad \forall\, i < j,\ i,j \in \{1,\dots,r\} \qquad (17)$$

Various relaxations are available [20, 21], in order to obtain less conservative stability conditions.

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Proof:

Let us consider the TS model (13) and the fuzzy observer (14). The following notations are defined: $\tilde{x}(t)=x(t)-\hat{x}(t)$ and $\tilde{\theta}(t)=\theta(t)-\hat{\theta}(t)$ are respectively the error estimation for the state model and for the unknown vector and we need to find the gains L_i and K_i for $i\in\{1,\dots,r\}$ such that $\tilde{x}(t)$ and $\tilde{\theta}(t)$ converge asymptotically to 0 as t go to ∞. The dynamics of these errors estimation can be written taking into account the n times derivative of $\hat{\theta}(t)$ in this form:

$$\begin{pmatrix} \dot{\tilde{x}}(t) \\ \dot{\tilde{\theta}}(t) \\ \vdots \\ \tilde{\theta}^{(n)}(t) \end{pmatrix} = \begin{pmatrix} A_z - L_z C_z & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -K_z & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}(t) \\ \tilde{\theta}(t) \\ \vdots \\ \tilde{\theta}^{(n-1)}(t) \end{pmatrix} \quad (18)$$

The stability of the observer (14) is studied with the second Lyapunov method [22] where the following quadratic Lyapunov function with a decay rate is chosen:

$$V\left(\tilde{x}(t),\tilde{\theta}(t)\right)=\begin{pmatrix}\tilde{x}\\ \tilde{\theta}\end{pmatrix}^T P\begin{pmatrix}\tilde{x}\\ \tilde{\theta}\end{pmatrix},\ P=P^T>0 \quad (19)$$

$$\dot{V}\left(\tilde{x}(t),\tilde{\theta}(t)\right)<-2\alpha V\left(\tilde{x}(t),\tilde{\theta}(t)\right) \quad (20)$$

Therefore, the estimation error is globally asymptotically stable with a decay rate α if the inequality (21) is satisfied:

$$\dot{V}\left(\tilde{x},\tilde{\theta}\right)+2\alpha V\left(\tilde{x},\tilde{\theta}\right)<0 \quad (21)$$

which leads to:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i h_j\begin{pmatrix}\tilde{x}\\ \tilde{\theta}\end{pmatrix}^T\left(\bar{A}_{ij}P+P\bar{A}_{ij}^{\ T}+2\alpha P\right)\begin{pmatrix}\tilde{x}\\ \tilde{\theta}\end{pmatrix}<0 \quad (22)$$

with $\bar{A}_{ij}=\begin{bmatrix} A_z - L_z C_z & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -K_z & 0 & 0 & \cdots & 0 \end{bmatrix}$.

And in order to find a solution to this problem, (22) is reformulated in the following form:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_i h_j\left[\left(P\begin{bmatrix} A_i & I & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}-P\begin{bmatrix}L_j\\ K_j\end{bmatrix}\begin{bmatrix}C_i & 0\end{bmatrix}\right) + (*) + 2\alpha P\right]<0 \quad (23)$$

Then, using the convex property of the membership function, the proof is over.

B. Algorithm of AFR estimation

The basic idea is to compute from the estimated $\hat{\theta}(t)$ the AFR of the cylinder i on the interval where only the mass flow coming from this cylinder is non zero. As an example, the AFR ratio for the cylinder 1 is given by:

$$\hat{\lambda}_1=\begin{cases}\dfrac{\sum_{i=1}^{n_{cyl}} d_i\hat{\lambda}_i}{d_1}, \text{ if } d_1\neq 0 \text{ and } d_2=d_3=d_4=0 \\ \text{previous } \hat{\lambda}_1, \text{otherwise}\end{cases} \quad (24)$$

IV. SIMULATION RESULTS

Several trials have been made using Matlab/Simulink to show the efficiency of the proposed method.
Applying of theorem 1 for $n=2$ gives:
The observation gain for the state vector :

$$L_2=L_6=\begin{bmatrix}-0.003 & 0.000\\ -0.002 & 0.3269\end{bmatrix}\times 10^3,\ L_3=L_7=\begin{bmatrix}-0.002 & 0.011\\ -0.031 & 0.2175\end{bmatrix}\times 10^3$$

$$L_4=L_8=\begin{bmatrix}-0.003 & 0.001\\ -0.002 & 0.3263\end{bmatrix}\times 10^3.$$

and the observation gain for the unknown vector :

$$K_1=K_3=K_5=K_7=\begin{bmatrix}0.001 & -0.002\\ 0.000 & -0.0514\\ 0.0027 & -0.000\\ 0.000 & -2.520\end{bmatrix}\times 10^3,$$

$$K_2=K_4=K_6=K_8=\begin{bmatrix}0.003 & 0.000\\ 0.001 & -0.0233\\ 0.0001 & -0.001\\ 0.0027 & -3.520\end{bmatrix}\times 10^3.$$

Figures 2 and 3 present a trial at 1500 rpm with zero as initial conditions for the observer.

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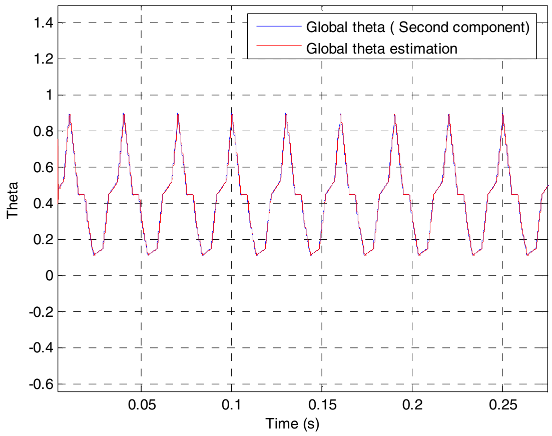

Figure 2: Evolution of theta and its estimate

The figure 2 shows the evolution of the second part of the vector $\theta(t)$ compared to its estimate $\hat{\theta}(t)$ and the figure 3 presents the associated error. This last remains less than 6% on this trial. Then, applying the algorithm related to equation (24) the estimated AFR for each cylinder is obtained (figure 5) to compare with the references (figure 4).

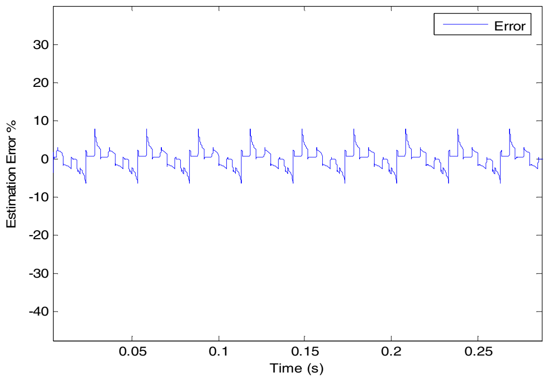

Figure 3: Estimation error (%)

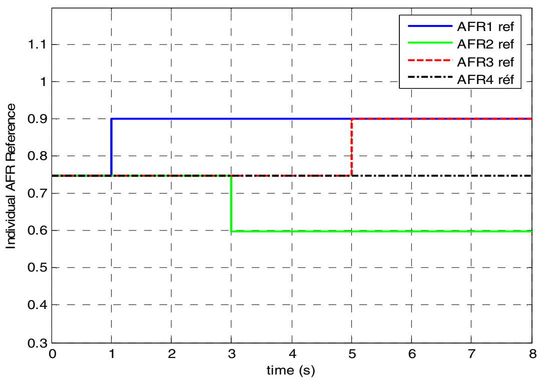

Figure 4: AFR reference for each cylinder

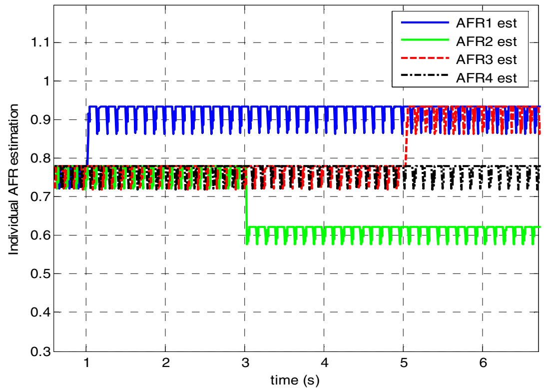

Figure 5: Estimated AFR for each cylinder

V. CONCLUSIONS

The purpose of this work was to develop a method to estimate the AFR of each cylinder. Based on the use a TS fuzzy observer, a first estimation of the state vector and of the unknown parameters is done. The main advantage of this design is its systematic aspect to deal with a large class of nonlinear systems. Then, from this estimation and the knowledge of the gas mass flow coming from each cylinder, an algorithm is developed to compute the AFR of each cylinder. The simulation results show the efficiency of the proposed method. Future works are concerning the control of the individual AFR based on the proposed approach for the estimation.

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