Switching Lyapunov functions for periodic TS systems

Zs. Lendek *,** **J. Lauber** * **T. M. Guerra** *

* *University of Valenciennes and Hainaut-Cambresis, LAMIH, Le Mont Houy, 59313 Valenciennes Cedex 9, France, (email: {jimmy.lauber, thierry.guerra}@univ-valenciennes.fr)*
** *Department of Automation, Technical University of Cluj-Napoca, Memorandumului 28, 400114 Cluj-Napoca, Romania, (email: zsofia.lendek@aut.utcluj.ro)*

Abstract: This paper considers the stability analysis of periodic Takagi-Sugeno fuzzy models. For this we use a switching Lyapunov function defined at the time instants when the subsystems switch. Using the developed conditions we are able to prove the stability of periodic TS systems where the local models or even the subsystems are unstable. The application of the conditions is illustrated on numerical examples.

Keywords: Stability analysis, TS systems, periodic systems, non-quadratic Lyapunov function.

1. INTRODUCTION

Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno, 1985) are nonlinear, convex combinations of local linear models, and have the property that they are able to exactly represent a large class of nonlinear systems (Lendek et al., 2010).

For the stability analysis and observer and controller design of TS systems the direct Lyapunov approach has been used. Stability conditions have been derived using quadratic Lyapunov functions (Tanaka et al., 1998; Tanaka and Wang, 2001; Sala et al., 2005), piecewise continuous Lyapunov functions (Johansson et al., 1999; Feng, 2004a), and more recently, to reduce the conservativeness of the conditions, nonquadratic Lyapunov functions (Guerra and Vermeiren, 2004; Kruszewski et al., 2008; Mozelli et al., 2009). The stability or design conditions are generally derived in the form of linear matrix inequalities (LMIs).

Non-quadratic Lyapunov functions have shown a real improvement of the design conditions in the discrete-time case (Guerra and Vermeiren, 2004; Ding et al., 2006; Dong and Yang, 2009a; Lee et al., 2011). It has been proven that the solutions obtained by non-quadratic Lyapunov functions include and extend the set of solutions obtained using the quadratic framework.

Non-quadratic Lyapunov functions have been extended to double-sum Lyapunov functions by (Ding et al., 2006) and later on to polynomial Lyapunov functions by (Sala and Ariño, 2007; Ding, 2010; Lee et al., 2010). A different type of improvement in the discrete case has been developed in (Kruszewski et al., 2008), conditions being obtained by replacing the classical one sample variation of the Lyapunov function by its variation over several samples (α-sample variation).

Switched TS systems are a class of nonlinear systems often described by continuous dynamics and discrete dynamics as well as their interactions. In the last decade, they have been investigated mainly in the continuous case where the stability is based on the use of a quadratic Lyapunov function (Tanaka et al., 2001; Lam et al., 2002, 2004; Ohtake et al., 2006) or a piecewise one (Feng, 2003, 2004b). Although results are available for discrete-time linear switching systems (Daafouz et al., 2002), for discrete-time TS models, few results exist (Doo et al., 2003; Dong and Yang, 2009b).

In this paper, we propose a switching non-quadratic Lyapunov function for the stability analysis of periodic TS fuzzy models. This Lyapunov function is useful for proving the stability of a periodic TS system having non-stable local models and even unstable subsystems. For simplicity, we present the results for a periodic system with two subsystems, although they can be easily generalized for a known number of subsystems.

The structure of the paper is as follows. Section 2 presents the notations used in this paper and the general form of the TS models. It also develops the proposed conditions for stability analysis of systems that switch at each sample time. The stability analysis of periodic systems is presented in Section 3. Section 4 illustrates the use of the conditions on a numerical example. Finally, Section 5 concludes the paper.

⋆ This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS – UEFISCDI, project number PN-II-RU-TE-2011-3-0043, contract number 74/05.10.2011, by International Campus on Safety and Intermodality in Transportation the European Community, the Délegation Régionale à la Recherche et à la Technologie, the Ministére de L'Enseignement supérieur et de la Recherche the Region Nord Pas de Calais and the Centre Nationale de la Recherche Scientifique: the authors gratefully acknowledge the support of these institutions.

2. STABILITY OF CONTINUOUSLY SWITCHING SYSTEMS

2.1 Preliminaries

In this paper we consider stability analysis of discrete-time periodic TS systems. For the ease of notation, we only consider two subsystems of the form

$$\begin{aligned}\boldsymbol{x}(T+T_1) &= \sum_{i=1}^{r_1} h_{1i}(\boldsymbol{z}_1(T))A_{1i}\boldsymbol{x}(T)\\ &= A_{1z}\boldsymbol{x}(T)\end{aligned} \tag{1}$$

and

$$\begin{aligned}\boldsymbol{x}(T+T_2) &= \sum_{i=1}^{r_2} h_{2i}(\boldsymbol{z}_2(T))A_{2i}\boldsymbol{x}(T)\\ &= A_{2z}\boldsymbol{x}(T)\end{aligned} \tag{2}$$

where $\boldsymbol{x}$ denotes the state vector, T is the current time, T_1 and T_2 are the sampling periods of the subsystems (i.e., the two subsystems may have different sampling times), r_1 and r_2 are the number of rules, $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are the scheduling vectors, h_{1i}, $i = 1, 2, \ldots, r_1$ and h_{2i}, $i = 1, 2, \ldots, r_2$ normalized membership functions, and A_{1i}, $i = 1, 2, \ldots, r_1$ and A_{2i}, $i = 1, 2, \ldots, r_2$ the local models.

We consider periodic systems, i.e., the two subsystems defined above are activated is a sequence $\underbrace{1, 1, \ldots, 1}_{p_1}$, $\underbrace{2, 2, \ldots, 2}_{p_2}$, $\underbrace{1, 1, \ldots, 1}_{p_1}$, etc., where p_1 and p_2 denote the periods of the subsystems.

In what follows, 0 and I denote the zero and identity matrices of appropriate dimensions, and a $(*)$ denotes the term induced by symmetry. The subscript $z+1$ (as in A_{1z+1}) stands for the scheduling vector evaluated at the next sampling time. Note that depending on the subsystem, the next sampling instant may be $T + T_1$ or $T + T_2$.

To derive stability conditions for periodic systems, we will make use of the following result:

Lemma 1. (Skelton et al., 1998) Consider a vector $\boldsymbol{x} \in \mathbb{R}^{n_x}$ and two matrices $Q = Q^T \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{m \times n_x}$ such that $\operatorname{rank}(R) < n_x$. The two following expressions are equivalent:

(1) $\boldsymbol{x}^T Q \boldsymbol{x} < 0$, $\boldsymbol{x} \in \{\boldsymbol{x} \in \mathbb{R}^{n_x}, \boldsymbol{x} \neq 0, R\boldsymbol{x} = 0\}$
(2) $\exists H \in \mathbb{R}^{m \times n_x}$ such that $Q + HR + R^T H^T < 0$

2.2 Stability analysis

In this section, we consider the simplest case when the two subsystems defined in Section 2.1 are switched at every sampling time, i.e., $p_1 = p_2 = 1$. For such systems, the following results can be stated:

Theorem 1. The periodic TS system composed of the subsystems (1) and (2), with periods $p_1 = p_2 = 1$ is asymptotically stable, if there exist $P_{1i} = P_{1i}^T > 0$, M_{1i}, $i = 1, 2, \ldots, r_1$, $P_{2i} = P_{2i}^T > 0$, M_{2i}, $i = 1, 2, \ldots, r_2$, so that the following conditions are satisfied:

$$\begin{aligned}&\begin{pmatrix} -P_{1z} & (*) \\ M_{2z}A_{2z} & -M_{2z} - M_{2z}^T + P_{2z+1}\end{pmatrix} < 0\\ &\begin{pmatrix} -P_{2z} & (*) \\ M_{1z}A_{1z} & -M_{1z} - M_{1z}^T + P_{1z+1}\end{pmatrix} < 0\end{aligned} \tag{3}$$

Proof: Consider the switching Lyapunov function, similar to the one used by Daafouz et al. (2002),

$$V(\boldsymbol{x}(T), T) = \begin{cases} \boldsymbol{x}(T)^T P_{1z}\boldsymbol{x}(T) & \text{active subsystem was 1} \\ \boldsymbol{x}(T)^T P_{2z}\boldsymbol{x}(T) & \text{otherwise}\end{cases}$$

Then the difference in the Lyapunov function is either

$$V(\boldsymbol{x}(T+T_2), T+T_2) - V(\boldsymbol{x}(T), T) = \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_2)\end{pmatrix}^T \begin{pmatrix} -P_{1z} & 0 \\ 0 & P_{2z+1}\end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_2)\end{pmatrix}$$

if the switching is from the first subsystem to the second one (Case 1), or

$$V(\boldsymbol{x}(T+T_1), T+T_1) - V(\boldsymbol{x}(T), T) = \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_1)\end{pmatrix}^T \begin{pmatrix} -P_{2z} & 0 \\ 0 & P_{1z+1}\end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_1)\end{pmatrix}$$

if the switching is from the second subsystem to the first one (Case 2).

Consider first Case 1. We have

$$\Delta V = \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_2)\end{pmatrix}^T \begin{pmatrix} -P_{1z} & 0 \\ 0 & P_{2z+1}\end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_2)\end{pmatrix}$$

together with the system dynamics, which is

$$(A_{2z} \; -I)\begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_2)\end{pmatrix} = 0$$

since during the time $[T, T+T_2]$, the second subsystem is active. Using Lemma 1, the difference in the Lyapunov function is negative definite, if there exists H such that

$$\begin{pmatrix} -P_{1z} & 0 \\ 0 & P_{2z+1}\end{pmatrix} + H\,(A_{2z} \; -I) + (*) < 0$$

Choosing $H = \begin{pmatrix} 0 \\ M_{2z}\end{pmatrix}$ leads directly to

$$\begin{pmatrix} -P_{1z} & (*) \\ M_{2z}A_{2z} & -M_{2z} - M_{2z}^T + P_{2z+1}\end{pmatrix} < 0$$

For Case 2, we have

$$\Delta V = \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_1)\end{pmatrix}^T \begin{pmatrix} -P_{2z} & 0 \\ 0 & P_{1z+1}\end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_1)\end{pmatrix}$$

and the dynamics

$$(A_{1z} \; -I)\begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T+T_1)\end{pmatrix} = 0$$

which, by choosing $H = \begin{pmatrix} 0 \\ M_{1z}\end{pmatrix}$ leads to

$$\begin{pmatrix} -P_{2z} & (*) \\ M_{1z}A_{1z} & -M_{1z} - M_{1z}^T + P_{1z+1}\end{pmatrix} < 0 \qquad \square$$

3. STABILITY ANALYSIS OF PERIODIC SYSTEMS

In the previous section, we considered the special case when the subsystems are switching at each sample time. Consider now the case when the subsystems switch after p_1, respectively p_2 samples. Then, the following result can be stated.

Theorem 2. The periodic TS system composed of the subsystems (1) and (2), with periods $p_1 \geq 1$ and $p_2 \geq 1$ is asymptotically stable, if there exist $P_{1i} = P_{1i}^T > 0$, M_{1i}, $i = 1, 2, \ldots, r_1$, $P_{2i} = P_{2i}^T > 0$, M_{2i}, $i = 1, 2, \ldots, r_2$, so that the following conditions are satisfied:

$$\begin{pmatrix} -P_{1z} & (*) & \ldots & (*) \\ M_{2z}A_{2z} & -M_{2z} - M_{2z}^T & \ldots & (*) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & M_{2z+p_2-1}A_{2z+p_2-1} & \Omega_{2z+p_2} \end{pmatrix} < 0$$
$$\begin{pmatrix} -P_{2z} & (*) & \ldots & (*) \\ M_{1z}A_{1z} & -M_{1z} - M_{1z}^T & \ldots & (*) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & M_{1z+p_1-1}A_{1z+p_1-1} & \Omega_{1z+p_1} \end{pmatrix} < 0 \tag{4}$$

where $\Omega_{1z+p_1} = -M_{1z+p_1-1} - M_{1z+p_1-1}^T + P_{1z+p_1}$, and $\Omega_{2z+p_2} = -M_{2z+p_2-1} - M_{2z+p_2-1}^T + P_{2z+p_2}$, and the subscript $z + \alpha$ denotes the scheduling vector being evaluated at time $T + \alpha T_1$ or $T + \alpha T_2$, depending on which subsystem is active.

Proof: Consider again the switching Lyapunov function

$$V(\boldsymbol{x}(T), T) = \begin{cases} \boldsymbol{x}(T)^T P_{1z}\boldsymbol{x}(T) & \text{active subsystem was 1} \\ \boldsymbol{x}(T)^T P_{2z}\boldsymbol{x}(T) & \text{otherwise} \end{cases}$$

defined only in the time instants when the system dynamics switches from one subsystem to another.

Then the difference in the Lyapunov function is either

$$V(\boldsymbol{x}(T + p_2T_2), T + p_2T_2) - V(\boldsymbol{x}(T), T) =$$
$$\begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_2T_2) \end{pmatrix}^T \begin{pmatrix} -P_{1z} & 0 \\ 0 & P_{2z+p_2} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_2T_2) \end{pmatrix}$$

if the switching is from the first subsystem to the second one (Case 1), or

$$V(\boldsymbol{x}(T + p_1T_1), T + p_1T_1) - V(\boldsymbol{x}(T), T) =$$
$$\begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_1T_1) \end{pmatrix}^T \begin{pmatrix} -P_{2z} & 0 \\ 0 & P_{1z+p_1} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_1T_1) \end{pmatrix}$$

if the switching is from the second subsystem to the first one (Case 2).

Consider first Case 1. We have

$$\Delta V = \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_2T_2) \end{pmatrix}^T \begin{pmatrix} -P_{1z} & 0 \\ 0 & P_{2z+p_2} \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \boldsymbol{x}(T + p_2T_2) \end{pmatrix}$$

together with the system dynamics, which is

$$\begin{pmatrix} A_{2z} & -I & 0 & \ldots & 0 \\ 0 & A_{2z+1} & -I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \vdots \\ \boldsymbol{x}(T + p_2T_2) \end{pmatrix} = 0$$

since during the time $[T, T + p_2T_2]$, the second subsystem is active. Using Lemma 1, the difference in the Lyapunov function is negative definite, if there exists H such that

$$\begin{pmatrix} -P_{1z} & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & P_{2z+p_2} \end{pmatrix} + H \begin{pmatrix} A_{2z} & -I & 0 & \ldots & 0 \\ 0 & A_{2z+1} & -I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -I \end{pmatrix} + (*) < 0$$

Choosing

$$H = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ M_{2z} & 0 & \ldots & 0 \\ 0 & M_{2z+1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & M_{2z+p_2-1} \end{pmatrix}$$

leads directly to

$$\begin{pmatrix} -P_{1z} & (*) & \ldots & (*) \\ M_{2z}A_{2z} & -M_{2z} - M_{2z}^T & \ldots & (*) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & M_{2z+p_2-1}A_{2z+p_2-1} & \Omega_{2z+p_2} \end{pmatrix} < 0$$

For Case 2, we have the dynamics

$$\begin{pmatrix} A_{1z} & -I & 0 & \ldots & 0 \\ 0 & A_{1z+1} & -I & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & -I \end{pmatrix} \begin{pmatrix} \boldsymbol{x}(T) \\ \vdots \\ \boldsymbol{x}(T + p_1T_1) \end{pmatrix} = 0$$

and by choosing

$$H = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ M_{1z} & 0 & \ldots & 0 \\ 0 & M_{1z+1} & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & M_{1z+p_1-1} \end{pmatrix}$$

we obtain

$$\begin{pmatrix} -P_{2z} & (*) & \ldots & (*) \\ M_{1z}A_{1z} & -M_{1z} - M_{1z}^T & \ldots & (*) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & M_{1z+p_1-1}A_{1z+p_1-1} & \Omega_{1z+p_1} \end{pmatrix} < 0$$

□

Remarks:

(1) The conditions of Theorems 1 and 2 can easily be transformed into LMIs, and the relaxations of (Wang et al., 1996) or (Tuan et al., 2001) can be used.
(2) When developing the conditions we exploited the fact that the subsystems switch in finite time. If the system can remain in one mode (i.e., one of the subsystems can be continuously active), the developed conditions are not sufficient to guarantee stability of the whole system. However, similar conditions can be derived.
(3) Also due to the fact that the subsystems switch in finite time, it is not necessary that the local models of the TS subsystems or even the subsystems themselves to be stable, as it will be illustrated in the next section. Indeed, neither the conditions of Theorem 1, nor those of Theorem 2 require the subsystems to be stable.
(4) Although the conditions of Theorems 1 and 2 concern only two subsystems, they can be easily extended to a fixed number of subsystems, leading to the conditions: *the periodic system with n subsystems, each having period p_i is asymptotically stable, if there exist $P_{ij} = P_{ij}^T > 0$, M_{ij}, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, r_i$ so that*

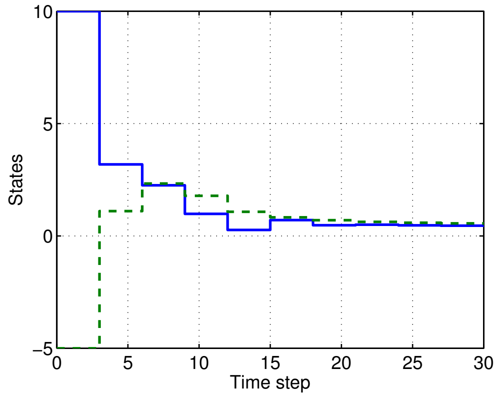

Fig. 1. Trajectories of the second subsystem – example 1.

$$\begin{pmatrix} -P_{iz} & (*) & \dots & (*) \\ \Gamma_{i0} & -M_{i+1z} - M_{i+1z}^T & \dots & (*) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \Gamma_{i,p_{i+1}} & \Omega_{i+1z+p_{i+1}} \end{pmatrix} < 0 \tag{5}$$

with $\Gamma_{ij} = M_{i+1z+j-1}A_{i+1z+j-1}$, $i = 1, 2, \dots, n$, $j = 0, 1, \dots, p_{i+1}$, $\Omega_{i+1z+p_{i+1}} = -M_{1z+p_{i+1}-1} - M_{1z+p_{i+1}-1}^T + P_{1z+p_{i+1}}$ *and the* $n+1$*th subsystem by definition being the first one.*

4. EXAMPLES

In this section we illustrate the use of the developed conditions on numerical examples.

Consider the switching fuzzy system with two subsystems as follows:

$$\boldsymbol{x}(T+T_1) = \sum_{i=1}^{2} h_{1i}(\boldsymbol{z}_1(T))A_{1i}\boldsymbol{x}(T)$$

with

$$A_{11} = \begin{pmatrix} -0.16 & -0.1 \\ 0.4 & 0.7 \end{pmatrix} \quad A_{12} = \begin{pmatrix} -1.1 & -0.15 \\ -0.67 & 0.24 \end{pmatrix}$$

with $T_1 = 1$, h_{11} randomly generated in $[0, 1]$, $h_{12} = 1 - h_{11}$ and

$$\boldsymbol{x}(T+T_2) = \sum_{i=1}^{2} h_{2i}(\boldsymbol{z}_1(T))A_{2i}\boldsymbol{x}(T)$$

with

$$A_{21} = \begin{pmatrix} 0.5 & 0.6 \\ 0.5 & 0.67 \end{pmatrix} \quad A_{22} = \begin{pmatrix} 0.4 & -0.4 \\ 0.16 & 0.36 \end{pmatrix}$$

with $T_2 = 3$, $h_{21} = cos(x_1)^2$, $h_{22} = 1 - h_{21}$.

Note that both A_{12} and A_{21} are unstable, their eigenvalues being $(-1.1712\ 0.3112)$ and $(0.0307\ 1.1393)$, respectively. The stability of the first subsystem depends on the exact values of the membership functions, while the second subsystem is locally stable, but not locally asymptotically stable (see Figure 1). Moreover, since the local models are unstable, this means that existing methods from the literature cannot be applied to prove the stability of this switching system.

However, by switching between the two subsystems at every time step, the whole system is asymptotically stable, as illustrated in Figure 2.

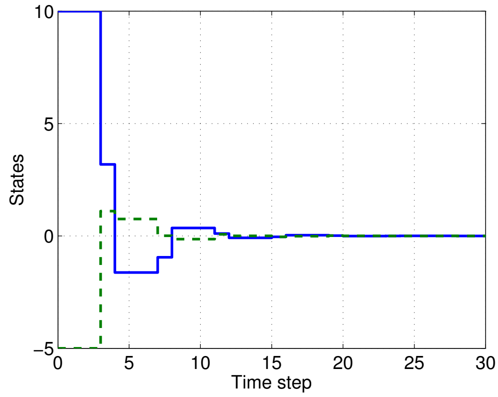

Fig. 2. Trajectories of the switching system – example 1.

Note that for this switching system it is not possible to find either a quadratic or a nonquadratic Lyapunov function, as the corresponding LMIs become unfeasible.

According to the proposed approach, we consider the switching Lyapunov function

$$V(\boldsymbol{x}(T), T) = \begin{cases} \boldsymbol{x}(T)^T P_{1z}\boldsymbol{x}(T) & \text{active subsystem was 1} \\ \boldsymbol{x}(T)^T P_{2z}\boldsymbol{x}(T) & \text{otherwise} \end{cases}$$

with $P_{1z} = h_{11}(T)P_{11} + h_{12}(T)P_{12}$, $P_{2z} = h_{21}(T)P_{21} + h_{22}(T)P_{22}$.

The LMIs corresponding to the conditions of Theorem 1, when using the relaxation of Wang et al. (1996) are

$$\begin{aligned} &\Gamma_{iik} < 0 \\ &\Gamma_{ijk} + \Gamma_{jik} < 0 \end{aligned} \tag{6}$$

where

$$\Gamma_{ijk} = \begin{pmatrix} -P_{1i} & (*) \\ M_{2i}A_{2j} & -M_{2i} - M_{2i}^T + P_{2k} \end{pmatrix}$$

or

$$\Gamma_{ijk} = \begin{pmatrix} -P_{2i} & (*) \\ M_{1i}A_{1j} & -M_{1i} - M_{1i}^T + P_{1k} \end{pmatrix}$$

By solving [1] the conditions (6), we obtain

$$P_{11} = \begin{pmatrix} 4.4614 & 2.3067 \\ 2.3067 & 7.1567 \end{pmatrix} \quad P_{12} = \begin{pmatrix} 5.0813 & -0.6803 \\ -0.6803 & 6.0733 \end{pmatrix}$$

$$M_{11} = \begin{pmatrix} 6.6673 & -0.7481 \\ 1.2718 & 6.9281 \end{pmatrix} \quad M_{12} = \begin{pmatrix} 6.6673 & -0.7481 \\ 1.2718 & 6.9281 \end{pmatrix}$$

$$P_{21} = \begin{pmatrix} 8.0167 & -1.8734 \\ -1.8734 & 6.5859 \end{pmatrix} \quad P_{22} = \begin{pmatrix} 13.3010 & -2.7835 \\ -2.7835 & 4.3840 \end{pmatrix}$$

$$M_{21} = \begin{pmatrix} 9.8200 & -3.7586 \\ -0.6206 & 6.9846 \end{pmatrix} \quad M_{22} = \begin{pmatrix} 10.86 & -1.744 \\ -1.993 & 6.46 \end{pmatrix}$$

and thereby prove the stability of the switching system.

To illustrate the conditions of Theorem 2, consider the periodic fuzzy system with two subsystems as follows:

$$\boldsymbol{x}(T+T_1) = \sum_{i=1}^{2} h_{1i}(\boldsymbol{z}_1(T))A_{1i}\boldsymbol{x}(T)$$

with

$$A_{11} = \begin{pmatrix} -0.44 & -0.26 \\ -0.65 & 0.62 \end{pmatrix} \quad A_{12} = \begin{pmatrix} 1.1 & -0.2 \\ 0.53 & -0.27 \end{pmatrix}$$

[1] For solving LMIs, the *feasp* function of Matlab has been used, with the default options.

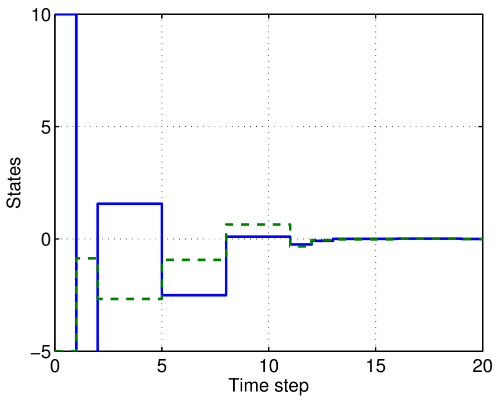

Fig. 3. Trajectories of the switching system – example 2.

with $T_1 = 1$, h_{11} randomly generated in $[0, 1]$, $h_{12} = 1 - h_{11}$ and

$$\boldsymbol{x}(T + T_2) = \sum_{i=1}^{2} h_{2i}(\boldsymbol{z}_1(T)) A_{2i} \boldsymbol{x}(T)$$

with

$$A_{21} = \begin{pmatrix} 0.02 & 0.6 \\ -0.22 & -0.44 \end{pmatrix} \quad A_{22} = \begin{pmatrix} 0.32 & -0.15 \\ -1 & 0.8 \end{pmatrix}$$

with $T_2 = 3$, $h_{21} = cos(x_1)^2$, $h_{22} = 1 - h_{21}$.

The local models A_{12} and A_{22} are unstable, their eigenvalues being $(1.0177\ -0.1877)$ and $(0.1044\ 1.0156)$, respectively. Again, this means that existing results from the literature cannot be applied. By switching between the two subsystems with a period $p_1 = 2$ for the first subsystem and $p_2 = 3$ for the second subsystem, the resulting switching system is asymptotically stable, as illustrated in Figure 3.

For this switching system it is not possible to find either a quadratic or a nonquadratic Lyapunov function, as the corresponding LMIs become unfeasible.

The conditions of Theorem 2 concerning the first subsystem are (the LMIs are obtained similarly for the second subsystem)

$$\begin{pmatrix} -P_{2z} & (*) & (*) \\ M_{1z}A_{1z} & -M_{1z} - M_{1z}^T & (*) \\ 0 & M_{1z+1}A_{1z+1} & -M_{1z+1} - M_{1z+1}^T + P_{2z+2} \end{pmatrix} < 0$$

that is,

$$\sum_{i,j,k,l,m=1}^{2} \begin{pmatrix} -P_{2i} & (*) & (*) \\ M_{1i}A_{1j} & -M_{1i} - M_{1i}^T & (*) \\ 0 & M_{1j}A_{1k} & -M_{1j} - M_{1k}^T + P_{2m} \end{pmatrix} < 0$$

for which relaxations such as those in (Wang et al., 1996) or (Tuan et al., 2001) can be used. In this paper, we used the relaxations of (Wang et al., 1996), and obtained

$$P_{11} = P_{12} = \begin{pmatrix} 0.1984 & -0.0815 \\ -0.0815 & 0.2034 \end{pmatrix}$$

$$M_{11} = M_{12} = \begin{pmatrix} 0.1973 & -0.0711 \\ -0.0537 & 0.1662 \end{pmatrix}$$

$$P_{21} = P_{22} = \begin{pmatrix} 0.2395 & 0.0035 \\ 0.0035 & 0.0867 \end{pmatrix}$$

$$M_{21} = M_{22} = \begin{pmatrix} 0.2215 & 0.0139 \\ -0.0153 & 0.1214 \end{pmatrix}$$

Consequently, the periodic TS model is asymptotically stable.

5. CONCLUSIONS

In this paper we have developed conditions for the stability of periodic TS systems. For this, we used a switching Lyapunov function, defined in the points where the subsystems themselves switch. The conditions can guarantee the stability of periodic systems even if the local models or even the subsystems are unstable.

REFERENCES

Daafouz, J., Riedinger, P., and Iung, C. (2002). Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach. *IEEE Transactions on Automatic Control*, 47(11), 1883–1887.

Ding, B. (2010). Homogeneous polynomially nonquadratic stabilization of discrete-time Takagi-Sugeno systems via nonparallel distributed compensation law. *IEEE Transactions of Fuzzy Systems*, 18(5), 994–1000.

Ding, B., Sun, H., and Yang, P. (2006). Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form. *Automatica*, 42(3), 503–508.

Dong, J. and Yang, G. (2009a). Dynamic output feedback H_∞ control synthesis for discrete-time T-S fuzzy systems via switching fuzzy controllers. *Fuzzy Sets and Systems*, 160(19), 482–499.

Dong, J. and Yang, G. (2009b). H_∞ controller synthesis via switched PDC scheme for discrete-time T-S fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 17(3), 544–555.

Doo, J.C., Seung, S.L., and PooGyeon, P. (2003). Output-feedback control of discrete-time switching fuzzy system. In *Proceedings of the IEEE International Conference on Fuzzy Systems*, 441–446. St. Luis, MO, USA.

Feng, G. (2004a). Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions. *IEEE Transactions on Fuzzy Systems*, 12(1), 22–28.

Feng, G. (2003). Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions. *IEEE Transactions on Fuzzy Systems*, 11(5), 605–612.

Feng, G. (2004b). H_∞ controller design of fuzzy dynamic systems based on piecewise Lyapunov functions. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 34(1), 283–292.

Guerra, T.M. and Vermeiren, L. (2004). LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form. *Automatica*, 40(5), 823–829.

Johansson, M., Rantzer, A., and Arzen, K. (1999). Piecewise quadratic stability of fuzzy systems. *IEEE Transactions on Fuzzy Systems*, 7(6), 713–722.

Kruszewski, A., Wang, R., and Guerra, T.M. (2008). Nonquadratic stabilization conditions for a class of uncertain nonlinear discrete time TS fuzzy models: A new approach. *IEEE Transactions on Automatic Control*, 53(2), 606–611.

Lam, H.K., Leung, F.H.F., and Lee, Y.S. (2004). Design of a switching controller for nonlinear systems with unknown parameters based on a fuzzy logic approach. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 34(2), 1068–1074.

Lam, H.K., Leung, F.H.F., and Tam, P.K.S. (2002). A switching controller for uncertain nonlinear systems. *IEEE Control Systems Magazine*, 22(1), 1–14.

Lee, D.H., Park, J.B., and Joo, Y.H. (2010). Improvement on nonquadratic stabilization of discrete-time Takagi-Sugeno fuzzy systems: Multiple-parameterization approach. *IEEE Transactions of Fuzzy Systems*, 18(2), 425–429.

Lee, D.H., Park, J.B., and Joo, Y.H. (2011). Approaches to extended non-quadratic stability and stabilization conditions for discrete-time Takagi-Sugeno fuzzy systems. *Automatica*, 47(3), 534–538.

Lendek, Zs., Guerra, T.M., Babuška, R., and De Schutter, B. (2010). *Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models*, volume 262 of *Studies in Fuzziness and Soft Computing*. Springer Germany.

Mozelli, L.A., Palhares, R.M., Souza, F.O., and Mendes, E.M.A.M. (2009). Reducing conservativeness in recent stability conditions of TS fuzzy systems. *Automatica*, 45(6), 1580–1583.

Ohtake, H., Tanaka, K., and Wang, H.O. (2006). Switching fuzzy controller design based on switching Lyapunov function for a class of nonlinear systems. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 36(1), 13–23.

Sala, A., Guerra, T.M., and Babuška, R. (2005). Perspectives of fuzzy systems and control. *Fuzzy Sets and Systems*, 156(3), 432–444.

Sala, A. and Ariño, C. (2007). Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of Polya's theorem. *Fuzzy Sets and Systems*, 158(24), 2671–2686.

Skelton, R.E., Iwasaki, T., and Grigoriadis, K. (1998). *A Unified Approach to Linear Control Design*. Taylor & Francis.

Takagi, T. and Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, 15(1), 116–132.

Tanaka, K., Ikeda, T., and Wang, H. (1998). Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs. *IEEE Transactions on Fuzzy Systems*, 6(2), 250–265.

Tanaka, K., Iwasaki, M., and Wang, H.O. (2001). Switching control of an R/C hovercraft: stabilization and smooth switching. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 31(6), 853–863.

Tanaka, K. and Wang, H.O. (2001). *Fuzzy Control System Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, New York, NY, USA.

Tuan, H., Apkarian, P., Narikiyo, T., and Yamamoto, Y. (2001). Parameterized linear matrix inequality techniques in fuzzy control system design. *IEEE Transactions on Fuzzy Systems*, 9(2), 324–332.

Wang, H., Tanaka, K., and Griffin, M. (1996). An approach to fuzzy control of nonlinear systems: stability and design issues. *IEEE Transactions on Fuzzy Systems*, 4(1), 14–23.