Adaptive fuzzy observer and robust controller for a 2-DOF robot arm

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Motivation

**TS fuzzy models**
- rule base with linear or affine consequents
- well-established methods, conditions based on LMI
- unmodelled dynamics – adaptive observers

**Robot manipulators**
- great importance: manufacturing, handling, house-hold tasks
- human-friendly, compliant mechanical design
- new control solutions

This talk
Adaptive observer and robust controller for a 2-DOF robot arm
Outline

1. 2-DOF Robot arm
2. Adaptive observer
3. Robust controller
4. Estimation in closed-loop
5. Conclusions
1. 2-DOF Robot arm
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The arm

\[ M_R(\theta) \ddot{\theta} + C_R \dot{\theta} = \tau \]

\[
M_R(\theta) = \begin{bmatrix}
P_1 + P_2 + 2P_3 \cos \theta_2 & P_2 + P_3 \cos \theta_2 \\
P_2 + P_3 \cos \theta_2 & P_2 
\end{bmatrix}
\]

\[
C_R(\theta, \dot{\theta}) = \begin{bmatrix}
b_1 - P_3 \dot{\theta}_2 \sin \theta_2 & -P_3 (\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 \\
-P_3 \dot{\theta}_2 \sin \theta_2 & b_2 
\end{bmatrix}
\]
State-space

\[
\dot{x} = \begin{bmatrix}
-M_R^{-1}C_R & 0 \\
0 & 0
\end{bmatrix} x + \begin{bmatrix}
M_R^{-1}
0
\end{bmatrix} \tau
\]

\[
x = [\dot{\theta}_1 \dot{\theta}_2 \theta_1 \theta_2]^T
\]

\[
\dot{x} = A(\theta, \dot{\theta})x + B(\theta)\tau
\]

\[
A = \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22} \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

6 nonlinearities
State-space – simplification

\[ C_R = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \]

\[ \dot{x} = A(z_1, z_2)x + B(z_1, z_2)\tau \]

\[ A = \begin{bmatrix} P_2 b_1 z_1 & -P_2 b_2 z_1 - P_3 b_2 z_2 & 0 & 0 \\ -P_2 b_1 z_1 - P_3 b_1 z_2 & b_2 z_1 (P_1 + P_2) + 2P_3 b_2 z_2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ B = \begin{bmatrix} -P_2 k_m z_1 & k_m (P_2 z_1 + P_3 z_2) \\ k_m (P_2 z_1 + P_3 z_2) & -k_m (2P_3 z_2 + z_1 (P_1 + P_2)) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]

2 nonlinearities
Comparison

2-DOF Robot arm

Adaptive observer

Robust controller

Estimation in closed-loop

Conclusions

Comparison

Normalized input voltage

Angular position in degrees

Time in seconds
TS model

Sector nonlinearity

\[ z_1 = \frac{1}{P_3^2 \cos(\theta_2)^2 - P_1 P_2} \]
\[ z_2 = \frac{\cos(\theta_2)}{P_3^2 \cos(\theta_2)^2 - P_1 P_2} \]
\[ w_{11}(z_1) = \frac{z_{1\text{max}} - z_1}{z_{1\text{max}} - z_{1\text{min}}} \]
\[ w_{12}(z_1) = 1 - w_{11}(z_1) \]

\[ \dot{x} = \sum_{i=1}^{r} h_i(z)(A_i x + B_i u) \]
\[ y = C x \]

Uncertainties in the state matrices
1. 2-DOF Robot arm

2. Adaptive observer

3. Robust controller

4. Estimation in closed-loop

5. Conclusions
Adaptive observer

To estimate unmodelled dynamics

Model:
\[
\dot{x} = \sum_{i=1}^{r} h_i(z)(A_i x + B_i u + M_i A_{\delta i} x)
\]
\[
y = C x
\]

Observer:
\[
\dot{\hat{x}} = \sum_{i=1}^{r} h_i(z)(A_i \hat{x} + B_i u + L_i(y - \hat{y}) + M_i(\hat{A}_{\delta i} \hat{x}))
\]
\[
\hat{y} = C \hat{x}
\]

Adaptive law:
\[
\dot{\hat{A}}_{\delta i} = h_i(z)M_i^T P C^\dagger e_y \hat{X}^T \quad i = 1, 2, \ldots, r
\]
Design

TS model of the robot arm: 4 rules

\[
\dot{x} = \sum_{i=1}^{r} h_i(z) (A_i x + B_i u + M_i A_{\delta i} x) \\
y = C x
\]

Uncertainty in the damping coefficients
Uncertainty distribution matrix:

\[
M_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

Uncertainty norm:

\[
\|A_{\delta i}\| \leq \mu_{\text{max}} = 2
\]
Results
Excitation
1. 2-DOF Robot arm
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Controller design

Model:
\[ \dot{x} = \sum_{i=1}^{r} h_i(z)(A_i x + B_i u + M_i A_{\delta i} x) \]

Control law:
\[ u = -\sum_{i=1}^{r} h_i(z) F_i x \]

Classical design conditions
1. 2-DOF Robot arm
2. Adaptive observer
3. Robust controller
4. Estimation in closed-loop
5. Conclusions
Results
Excitation

- $h_1$
- $h_2$
- $h_3$
- $h_4$

Time in seconds
1. 2-DOF Robot arm
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Conclusions

- adaptive observer for a 2-DOF robot arm
- estimation of the uncertainties in closed-loop

- exploiting the structure of the uncertainty
- convergence depending on the rule excitation
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